MATHEMATICS 31

LEARNING FACILITATOR'S MANUAL

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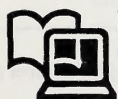
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UNITS 1 - 9

Distance Learning

Alberta EDUCATION

Mathematics 31

Units 1-9

LEARNING FACILITATOR'S MANUAL

Distance Learning

Alberta EDUCATION

Note

This Mathematics Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final tests until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

Contents

Contents

Introduction

A survey of these course materials will confirm that this new learning package has been specially designed for many kinds of teachers working in a variety of situations.

Which Category Do You Fit?

☐ Small Schools Teacher

    ☐ inexperienced

    ☐ experienced, but in other subject areas

    ☐ experienced in teaching Mathematics 31 but wanting to try a different approach

☐ Distance Learning Teacher

    ☐ travelling to schools within the jurisdiction

    ☐ using facsimile and teleconferencing to teach students within the area

☐ Larger Schools Teacher

    ☐ inexperienced

    ☐ experienced in teaching Mathematics 31 but wanting to try a different approach

Because these materials have been created by experienced classroom teachers and distance learning specialists, they have many advantages for students and teachers regardless of their situation.

Advantages for Students

- incorporates a strong learner-centred philosophy

- promotes such qualities in the learner as autonomy, independence, and flexibility

- is developed through media which suits the needs and circumstances of the learner

- reflects the experiential background of Alberta students

- opens up opportunities by overcoming barriers that result from geographical location

- promotes individualized learning, allowing learners to work at their own pace

Advantages for Teachers

- allows teachers maximum teaching time and minimizes preparation time

- includes different routes through the materials to suit different learners

- incorporates a wide range of teaching strategies, in particular those using independent and individual learning

- delivers curriculum designed by education specialists that reflects Alberta Education Program of Studies with an emphasis on Canadian content

- provides learning materials which are upwardly compatible with advanced educational technology

Does it sound like something you could use?

The student materials are not the only components designed for independent, guided instruction; so is this Learning Facilitator's Manual. It begins with an overview of the current Alberta Education Program of Studies for high school mathematics. This summary is included for inexperienced teachers or those teachers who have found themselves teaching high school mathematics when their training is in other subject areas. This brief summary is not meant to replace the Alberta Education Program of Studies, but rather to help teachers confirm the highlights of the program.

Other parts of this introduction have also been included to help teachers become familiar with this new courseware and determine how they might want to use it in their classroom.

Beyond the introduction the guide itself contains answers, models, explanations, and other tips generated by the teachers who authored this course.

The courseware and LFM are the products of experienced classroom teachers and distance learning specialists. It is the hope of these teachers that their experience can be shared with those who want to take advantage of it.
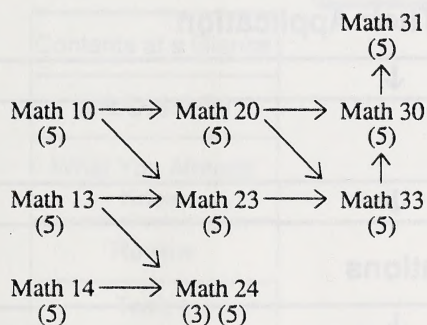
Overview of the Program of Studies

A knowledge of mathematics is essential in many other areas of learning. With available information and the use of technology the emphasis will not be so much on memorizing formulas but on how to use mathematical information to solve problems. The student will be expected to construct, investigate, translate, discuss, analyze, and formulate mathematics in a dynamic way.

The Mathematics 10 – 20 – 30 – 31 sequence is designed for students with an interest and aptitude in mathematics who are intending to pursue postsecondary studies at a university or in a mathematics-intensive program at a technical school or college. Having successfully completed Mathematics 30, students will have fulfilled the mathematics requirement for the Advanced High School Diploma.

The Mathematics 13 – 23 – 33 sequence is the mainstream mathematics program designed for students who require mathematics to prepare for universities, colleges, trades, and employment. The Mathematics 13 – 23 – 33 sequence will satisfy the requirements for the General High School Diploma.

The Mathematics 14 – 24 program is a general series suitable for students who have experienced difficulties in previous mathematics courses.

The complete high school mathematics program is illustrated below along with recommended transfer points. The number in brackets indicates credits.

		Math 31 (5)
Math 10 (5) →	Math 20 (5) →	Math 30 (5)
Math 13 (5) →	Math 23 (5) →	Math 33 (5)
Math 14 (5) →	Math 24 (3) (5)	

This mathematics course is divided into units. Each unit covers a major area of mathematical study, and is divided further into specific content areas called topics. Each unit has its own assignment booklet which the student completes and submits at the end of that unit.

Some of our distance learning courses are divided into sections within modules. If you receive letters or other materials from the Alberta Distance Learning Centre which refer to modules, please remember that a module is the same as a unit and a section is the same as a topic.

$$\text{Module} = \text{Unit} \qquad \text{Section} = \text{Topic}$$

Unit 1 Introduction to Differential Calculus

↓

Unit 2 Differentiation of Algebraic Expressions and Graphing

↓

Unit 3 Practical Application of Derivatives

↓

Unit 4 Integration

↓

Unit 5 Geometric Vectors and Their Application

↓

Unit 6 Algebraic Vectors and Their Application

↓

Unit 7 Inner Product

↓

Unit 8 Systems of Linear Equations

↓

Unit 9 Matrices and Linear Transformations
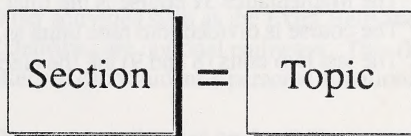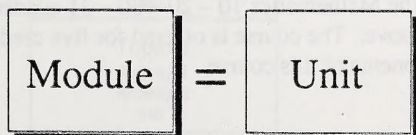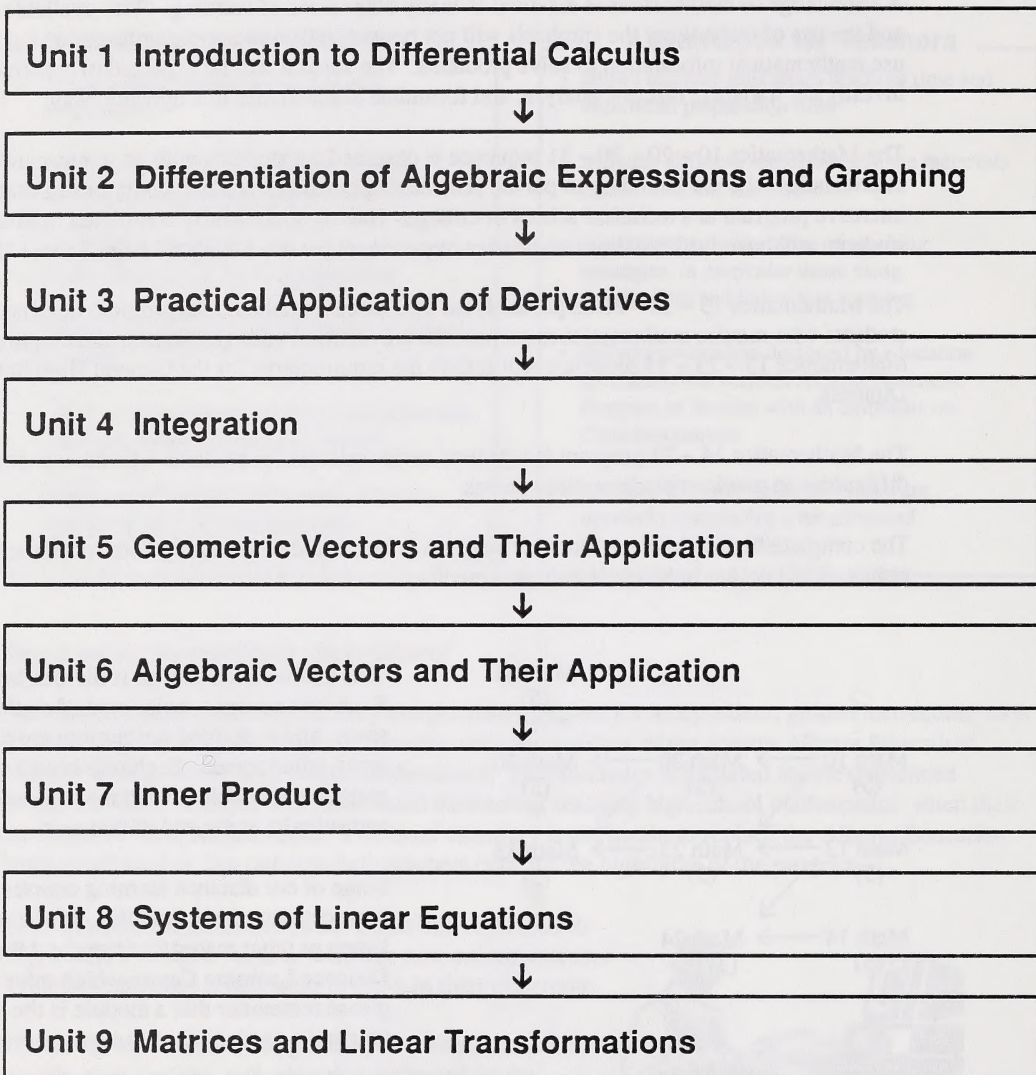
The Mathematics 31 course is the final course in the Mathematics 10 – 20 – 30 – 31 academic series. The course is divided into nine units as outlined above. The course is offered for five credits only. The last two units (8 and 9) are the elective components of this course.

Structure of the Learning Package

Basic Design

This new learning package involves many other components in addition to the Learning Facilitator's Manual.

Units

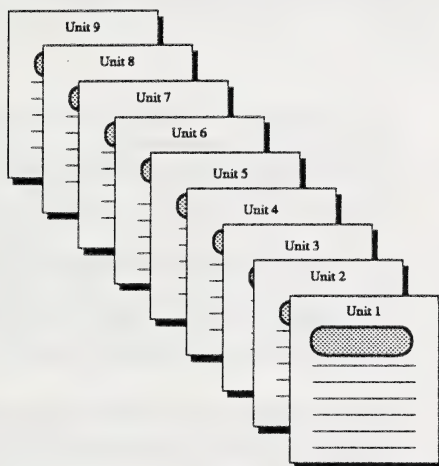| Contents at a Glance |
| :---: |
| Unit Overview |
| What You Already Know |
| Review |
| Topic 1
Activity 1
Activity 2
etc. |
| Topic 2
Activity 1
Activity 2
etc. |
| Topic 3
Activity 1
Activity 2
etc. |
| Unit Summary |
| Appendices |

The print components involve many booklets called units. These units contain guided activities that instruct students in a relevant, realistic setting.

The units have been specially designed to promote such qualities in the learner as autonomy, independence, and flexibility. Writers have incorporated such teaching strategies as working from the concrete to the abstract, linking the old to the new, getting students actively involved, and using advance, intermediate, and post organizers. Many other techniques enable learners to learn on their own for at least some of the time.

The structure of the unit booklets follows a systematic design. Each unit begins with a detailed table of contents which shows the students all the main steps. It acts as an organizer for students. The unit overview introduces the unit topic or theme. A graphic representation has been included to help visual learners and poor readers. The course overview states the weightings of each assignment.

The body of the unit is made up of two or more closely related topics. Each topic contains student activities that develop skills and knowledge centred around an objective.

The activities may involve print, audio, video, and computer formats. At times the student and the learning facilitator are allowed to choose the activity that best suits the student's needs and interests. Other activities such as the Extra Help and Extensions are optional pathways. This flexibility caters to each student's personal situation.

The summary focuses on the skills and strategies that the student has learned.

Assignment Booklet
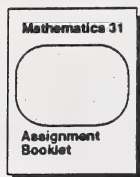
Accompanying each unit is an assignment booklet. The activities in these booklets can be used for formative and for summative assessments. The students should complete these assignment booklets when they have thoroughly reviewed the unit materials. The assignment booklets have been designed for classroom use, for faxing, or for mailing. **If the booklets are not being mailed, you should remove the outside cover.**

Media

COMPUTER DISK

VIDEOCASSETTE

AUDIOCASSETTE

The package also includes reference to media. Pathways have been developed so students can use a variety of media to achieve the objective. These different routes have been included to suit different learners. Wherever videos or computer software have been included, a print pathway is also available. This way, if the media resource isn't available or desired, a student can follow the print pathway and still successfully achieve the objective.

Special audiocassettes guide the student through difficult concepts. The appearance of the audiocassette icon reminds students that there is this additional help available.

If the students are working individually, you may find this cassette a valuable asset. If you are working in a large group, you may wish to guide the students yourself.

Materials, Media, and Equipment

Mandatory Components

Equipment (Hardware)	Media	Materials
• audiocassette player	• prepared audiocassettes (come with learning package)	• LFM for Mathematics 31 • one complete set of unit booklets (9) and assignment booklets (9) for each student • geometry set • scientific calculator • There is a final test.

Optional Components

Equipment (Hardware)	Media	Materials
• VCR	• videocassettes Videocassettes used in the course may be available from the Learning Resources Distributing Centre or ACCESS Network. You may also wish to call your regional library service for more information.	

Using This Learning Package in the Classroom

Conventional Classroom

Whether your classroom has desks in rows or tables in small groups, you may be most comfortable with a learning system that you can use with all your students in a paced style. In other words, you may want a package that will suit all of your students, so they can move through the materials as one group or several small groups. Because these materials contain different routes or pathways within each unit, they can address various learning styles and preferences. The materials also include many choices within the activities to cater to different thinking levels and ability levels. Because of their versatility and flexibility, these materials can easily suit a conventional classroom.

Open-Learning Classroom

Open-learning is the concept of opening up opportunities by overcoming barriers of time, pace, and place by giving the learners a package specially designed to enable them to learn on their own for at least some of the time.

Such a concept is not new. Many teachers can recite attempts to establish an individualized learning system as they recognized the importance of trying to personalize courseware to meet each individual student's needs. But these efforts often failed, due to lack of time and lack of quality materials that conformed to Alberta specifications.

Due to advanced educational technology and improved Alberta-specific learning packages, a student-centred approach is now possible. Improved technology now allows us to provide support to learners, individually regardless of their pace or location. A teacher cannot be in twenty-eight places at one time offering guidance. However, media and a well-designed learning package can satisfy individual needs. Technology can also help provide an effective management system needed to track the students as they progress independently through the materials.

The key to a successful open-learning system depends on three vital elements: a learning package specially designed to enable students to learn effectively on their own for at least some of the time; various kinds of learner support; and a management system and style that ensures that the open-learning system runs smoothly.

The Key to a Successful Open-Learning System

Learning Package: Television, Books, Computer, Guided Instruction, Audio, Video, Teleconferencing

Support: Feedback, Direct Teaching, Counselling, Routine Contact, Practical or Hands-on Experience

Management: Scheduling, Distributing, and Managing Resources, Monitoring and Recording Student Progress, Scheduling Students

Learning Package

The specially designed learning package needed for a successful open-learning system has been developed for you. The objectives teach current Alberta specifications using strategies designed for individualized instruction. As the learning facilitator, you need to be sure to have all the components in the learning package available to students as needed.

If adequate numbers of media are available to satisfy the demand, a centre can be established for specific media.
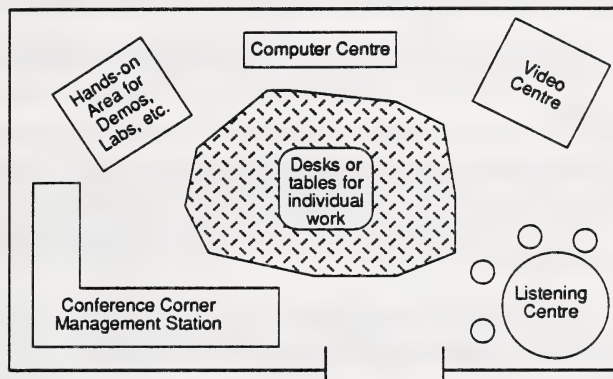
You may not have the luxury to have enough hardware to set up a permanent video or computer centre in your classroom. In that case, students should be encouraged to plan ahead. Perhaps every three to five days they should preview their materials and project when they would need a certain piece of media. This would allow you to group students, if necessary, or reserve media as required.

Support

Support is definitely a key element for successful learning, and when you're planning an individualized, non-paced program, you need to carefully plan when and how support will be given.

The materials contain a form of consistent support by providing immediate feedback for activities included in the unit booklet. High school students have solutions, models, explanations, and guides included in the appendix of every unit booklet. These are included so students can receive immediate feedback to clarify and reinforce their basic understanding before they move on to higher levels of thinking.

As the learning facilitator, you may be needed to offer more personal guidance to those students having difficulty, or you may need to reinforce the need for students to do these activities carefully before attempting the assignments in the assignment booklet.

The activities include choices and pathways. If a student is having difficulty, you may need to encourage that student to work on all the choices rather than one. This would provide additional instruction and practice in a variety of ways.

Another form of support is routine contact with each individual. This might be achieved with a biweekly conference scheduled by you, or as students reach a certain point (e.g., after each section is completed), they may be directed to come to the conference area.

Special counselling may be needed to help students through difficult stages. Praise and encouragement are important motivators, particularly for those students who are not used to working independently.

Direct teaching may be needed and scheduled at certain points in the program. This might involve small groups or a large group. It might be used to take advantage of something timely (e.g., election, eclipse, etc.) or something prescheduled like the demonstration of a process, or involving students in a hands-on, practical experience.

Support at a distance might include tutoring by phone, teleconferencing, faxing, or planned visits. These contacts are the lifeline between learners and distance education teachers, so a warm dialogue is essential.

Management

Good management of an open-learning system is essential to the success of the program. The following areas need action to ensure that the system runs smoothly:

- Scheduling, Distributing, and Managing Resources – As discussed earlier, this may require a need for centres or a system for students to project and reserve the necessary resources.

- Scheduling Students – Students and teachers should work together to establish goals, course completion timelines, and daily timelines. Although students may push to continue for long periods of time (e.g., all morning), teachers should discourage this. Concentration, retention, and motivation is improved by taking scheduled breaks.

- Monitoring Student Progress – You will need to record when units are completed by each student. Your data might also include the projected date of completion if you are using a student contract approach.
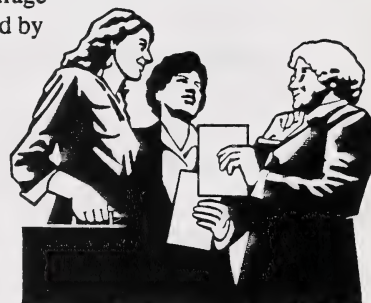
Sample of a Student Progress Chart

Mathematics 31		Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Unit 9	Final Test
Billy Adams	P										
	A										
Louise Despins	P										
	A										
Violet Klassian	P										
	A										
		P = Projected Completion Date				A = Actual Completion Date					

The student could keep a personal log as well. Such tracking of data could be stored easily on a computer.

- Recording Student Assessments – You will need to record the marks awarded to each student for work completed in each unit assignment booklet. The marks from these assignment booklets will contribute to a portion of the student's final mark. Other criteria may also be added (a special project, effort, attitude, etc.). Whatever the criteria, they should be made clear to all students at the beginning.

Sample of a Student's Assessment Chart

Mathematics 31	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Unit 9	Year's Average	Final Test	Final Mark
Billy Adams	67	65	54	47	78	67	63	77	65	65		
Louise Despins	43	50	54	55	48	42	52	41	47	48		
Violet Klassian	65	65	66	68	67	70	66	61	66	66		

Letter grading could easily be substituted.

- Recording Effectiveness of System – Keep ongoing records of how the system is working. This will help you in future planning.

Sample of a System Assessment Chart

Unit 1			
Date	Unit Booklet	Assignment Booklet	Resources/Media

The Role of the Teacher in an Open-Learning Classroom

The teachers in a conventional classroom spend a lot of time talking to large groups of learners. The situation in open learning requires a different emphasis. Teachers will probably meet learners individually or in very small groups.

With this approach it is necessary to move beyond the idea of a passive learner depending largely on a continually supportive teacher. The teacher must aim to build the student's confidence, to stimulate the learner into self-reliance, and to guide the learner to take advantage of routes that are most meaningful and applicable to the learner.

These materials are student-centred, not teacher-centred. The teacher needs to facilitate learning by providing general support to the learner.

Evaluation

Evaluation is important to the development of every learner. Data gathering and processing, and decision making, at the student and teacher level, serve as means of identifying strengths and weaknesses.

These specially designed learning packages contain many kinds of informal and formal evaluation.

Observation

In the classroom the teacher has the opportunity to see each student perform every day and to become aware of the level and nature of each student's performance.

Observations are more useful if they are recorded in an organized system. The following list of questions is a sample of types of observations and how they can be collected.

Observation Checklist

1. Does the student approach the work in a positive manner?

2. Is the student struggling with the reading level?

3. Does the student make good use of time?

4. Does the student apply an appropriate study method?

5. Can the student use references effectively, etc.?

Observation may suggest a need for an individual interview with a student.

Individual Conferences

Individual conferences may be paced (scheduled) by the calendar or at certain points in the unit, or they may be set up only as needed or requested.

During these conferences teachers can determine the student's progress and can assess the student's attitudes toward the subject, the program, school, and self, as well as the student's relationship with other students. With guided questions the teacher can encourage oral self-assessment; the student can discuss personal strengths or weaknesses in regard to the particular topic, unit, or subject area.

Self-Appraisal

Self-appraisal helps students recognize their own strengths and weaknesses. Through activities that require self-assessment, students also gain immediate feedback and clarification at early stages in the learning process. Teachers need to promote a responsible attitude toward these self-assessment activities. Becoming effective self-assessors is a crucial part of becoming an autonomous learner. By instructing, motivating, providing positive reinforcement, and systematically supervising, the learning facilitator will help students develop a positive attitude toward their own progress.

For variation, students may be paired and peer-assessing may become part of the system. The teacher may decide to have the student self-assess some of the activities, have a peer assess other activities, and become directly involved in assessing the remainder of the activities.

When the activities have been assessed, the student should be directed to make corrections. This should be made clear to students right from the start. It is important to note the correct association between the question and the response to clarify understanding, aid retention, and be of use for study purposes.

Many of the activities include choices for students. If the student is having difficulty, more practice may be warranted, and the student may need to be encouraged to do more of the choices.

Each topic within a unit includes additional types of activities called Extra Help and Extensions. Students are expected to be involved in the decision as to which pathway best suits their needs. They may decide to do both.

Self-appraisal techniques can also be introduced at the individual conferences. Such questions as the following might be included:

- What steps are you taking to improve your understanding of this topic?
- What method of study do you use most?
- How do you organize your material to remember it?
- What steps do you follow when doing an assignment in your assignment booklet?
- What could you do to become an even better reader?
- Do you have trouble following directions?
- Did you enjoy this unit?

A chart or checklist could be used for recording responses.

Informal Evaluation: Assignments

Informal evaluation, such as the assignments included in each unit, are an invaluable aid to the teacher. They offer ongoing assessment information about the student's achievement and the behaviour and attitudes that affect that achievement.

Each unit contains a separate booklet called the Assignment Booklet. This booklet assesses the knowledge or skills that the student has gained from the unit. **The student's mark for the unit may be based solely on the outcome of learning evident in the assignment booklet; however, you may decide to establish a value for other variables such as attitude or effort.** It is important that you establish at the beginning which outcomes will be evaluated and that all students clearly understand what is expected.

Final Test

All LFMs include a formal final test which can be photocopied for each member of the class. The test, closely linked to the learning outcomes stated in the unit booklets, give the teacher precise information concerning what each student can or cannot do. Answers and explanations and marking guides are also included.

The value of the final test and each unit is the decision of the classroom teacher. Following is a suggestion only.

Unit 1 5%	**Unit 2** 5%	**Unit 3** 10%
Unit 4 5%	**Unit 5** 5%	**Unit 6** 5%
Unit 7 5%	**Unit 8** 5%	**Unit 9** 5%
	Final Test 50%	

Introducing Students to the System

Your initiation to these learning materials began with a basic survey of what was included and how the components varied. This same process should be used with the class. After the materials have been explored, a discussion might include the advantages and the disadvantages of learning independently or in small groups. The roles of the students and teacher should be analyzed. The necessary progress checks and rules need to be addressed. Your introduction should motivate students and build a responsible attitude toward learning autonomously.

Skill Level

It is important for students to understand that there are certain skills that they will need in order to deal successfully with the course materials. They are listed below:

- understanding and using instructional materials (table of contents, index, list of illustrations, and appendices)

- interpreting graphs and charts

- using reference materials

- recognizing special symbols

- using a scientific calculator

Other general skills are using reliable study methods, outlining, and learning to read at a flexible rate.

To decide the level and amount of instruction needed to accommodate the varied levels among students, you may wish to prepare and administer skill inventories or pretests. If most students need help with a particular skill, you may want to plan a total class instructional session. If only certain students lack a skill, you may want to set up a temporary skill group to help students who need it, or you may want to develop a skills file for this purpose.
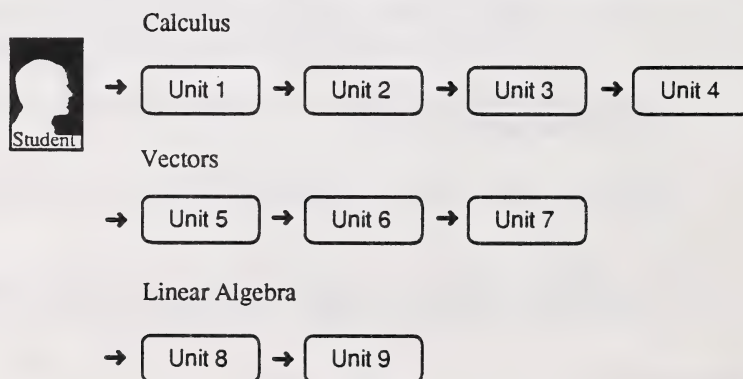
Reading Level

These course materials are largely print based, but poorer readers need not be discouraged. It is important that you assure the students that these materials have been designed for easy reading. The authors have employed special strategies that lower and control the reading level. Some of them are

- the conscious selection of vocabulary and careful structuring of sentences to keep the materials at an independent reading level

- the integration of activities, examples, and illustrations to break text into appropriate-sized chunks

- the inclusion of many kinds of organizers (advance, graphic, intermediate, concept mapping, post organizers) to help give students a structure for incorporating new concepts

- the recognition that vocabulary and concepts are basic to understanding content materials and thus, must be handled systematically (defined in context, marginal notes, footnotes, and often in a specialized glossary)

- the acknowledgement that background knowledge and experience play a vital role in comprehension

- the systematic inclusion of illustrations and optional videos to help poorer readers and visual learners, and audiocassettes and software as an alternative to print-based learning

- a variety of formats (paragraphs, lists, charts, etc.) to help poorer readers who do not absorb or retain main ideas easily in paragraph format

- the inclusion of media pathways and activity choices to encourage an active rather than passive approach

- instruction in a meaningful setting rather than in a contrived, workbook style

- using purposeful reading, viewing, and doing to produce better interpretation of the course materials

- the recognition that students need structured experiences when reading, viewing, or listening to instructional materials: developing pupil readiness, determining the purpose, providing guided instruction and feedback, rereading if necessary, and extending (This structure closely resembles the reading process.)
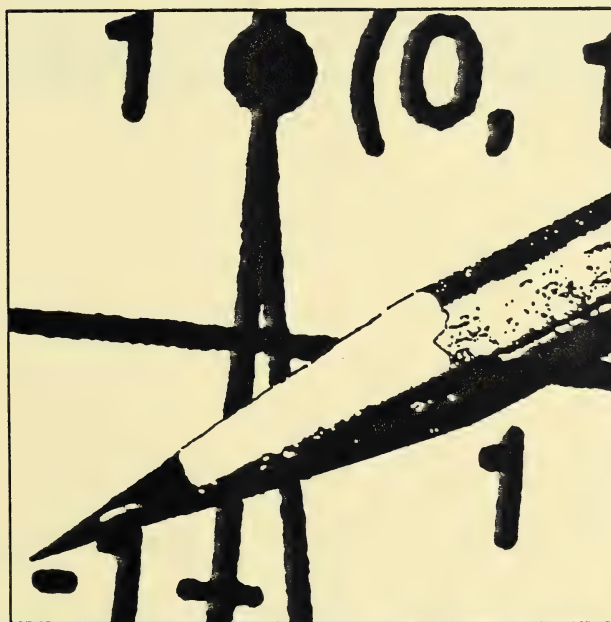
To help make the learning package more readable, you can begin in your unit preparation by reading (viewing, listening to) all the related materials that are going to be used. You need a solid background in order to assess and develop a background knowledge for students. The students' experiential bases may be assessed through brainstorming sessions concerning the topic, having students write down all the ideas they have about the topic, or by using visuals and guided questions to predict what the topic might be about.

It is recommended that you start with Unit 1 because this unit includes basic introductory information, and it is also recommended that you end with Unit 9.

Calculus

Student → Unit 1 → Unit 2 → Unit 3 → Unit 4

Vectors

→ Unit 5 → Unit 6 → Unit 7

Linear Algebra

→ Unit 8 → Unit 9

MATHEMATICS 31

LEARNING FACILITATOR'S MANUAL

UNIT 1: INTRODUCTION TO DIFFERENTIAL CALCULUS

Distance Learning

Alberta
EDUCATION

Note

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Topic 1: Limits and Derivatives

(2)   1. Determine the fifth term if $f(n) = (-3)^n$, $n \in N$.

$$f(5) = (-3)^5$$
$$= -243$$

(2)   2. Determine the sixth term if $f(n) = \dfrac{1}{4^n} - 2$, $n \in N$.

$$f(6) = \frac{1}{4^6} - 2$$

$$= \frac{1}{4096} - 2$$

$$= \frac{1 - 8192}{4096}$$

$$= \frac{-8191}{4096}$$

(5)   3. Give the first five terms of the sequence $f(n) = 3 - \dfrac{1}{n^2}$, $n \in N$.

$$f(1) = 3 - 1 \qquad\qquad f(2) = 3 - \frac{1}{2^2} \qquad\qquad f(3) = 3 - \frac{1}{3^2}$$

$$= 2 \qquad\qquad\qquad\qquad = 3 - \frac{1}{4} \qquad\qquad\qquad = 3 - \frac{1}{9}$$

$$\qquad\qquad\qquad\qquad\quad = 2\frac{3}{4} \qquad\qquad\qquad\quad = 2\frac{8}{9}$$

$$f(4) = 3 - \frac{1}{4^2} \qquad\qquad f(5) = 3 - \frac{1}{5^2}$$

$$= 3 - \frac{1}{16} \qquad\qquad\qquad = 3 - \frac{1}{25}$$

$$= 2\frac{15}{16} \qquad\qquad\qquad\quad = 2\frac{24}{25}$$

The first five terms are 2, $2\frac{3}{4}$, $2\frac{8}{9}$, $2\frac{15}{16}$, and $2\frac{24}{25}$.

③ 4. Find the limit L if $L = \lim\limits_{x \to 1}\left[\dfrac{8x-4}{2x-1}\right]$.

$$\lim\limits_{x \to 1}\left[\dfrac{8x-4}{2x-1}\right] = \lim\limits_{x \to 1}\dfrac{4\cancel{(2x-1)}}{\cancel{(2x-1)}}$$

$$= 4$$

③ 5. Find the limit L if $L = \lim\limits_{x \to -3}\left[\dfrac{x^2-x-12}{x+3}\right]$.

$$\lim\limits_{x \to -3}\left[\dfrac{x^2-x-12}{x+3}\right] = \lim\limits_{x \to -3}\dfrac{\cancel{(x+3)}(x-4)}{\cancel{(x+3)}}$$

$$= -3-4$$

$$= -7$$

6. Find the limit of each of the following.

(2)

a. $\lim\limits_{n\to\infty}\left[\dfrac{3}{n^2}\right]$

$\lim\limits_{n\to\infty}\dfrac{3}{n^2}=0$

(5)

b. $\lim\limits_{x\to\infty}\left[\dfrac{3x^2-5x+8}{x^2+2x-7}\right]$

$\lim\limits_{x\to\infty}\left[\dfrac{3x^2-5x+8}{x^2+2x-7}\right]=\lim\limits_{x\to\infty}\left[\dfrac{3-\dfrac{5}{x}+\dfrac{8}{x^2}}{1+\dfrac{2}{x}-\dfrac{7}{x^2}}\right]$

$\qquad\qquad =\dfrac{3}{1}$

$\qquad\qquad =3$

(5)

c. $\lim\limits_{x\to\infty}\left[\dfrac{x^3-x^2+x-5}{x^2+1}\right]$

$\lim\limits_{x\to\infty}\left[\dfrac{x^3-x^2+x-5}{x^2+1}\right]=\lim\limits_{x\to\infty}\left[\dfrac{1-\dfrac{1}{x}+\dfrac{1}{x^2}-\dfrac{5}{x^3}}{\dfrac{1}{x}+\dfrac{1}{x^3}}\right]$

$\qquad\qquad =\dfrac{1-0+0-0}{0+0}$

$\qquad\qquad =\dfrac{1}{0}$

$\qquad\qquad =\text{undefined}$

Therefore, there is no limit.

④ d. $\lim\limits_{x\to 2}\left[\dfrac{x^2+5x-14}{x-2}\right]$

$$\lim\limits_{x\to 2}\left[\dfrac{x^2+5x-14}{x-2}\right]=\lim\limits_{x\to 2}\dfrac{(x-2)(x+7)}{(x-2)}$$
$$=\lim\limits_{x\to 2}(x+7)$$
$$=9$$

⑥ e. $\lim\limits_{x\to 0}\left[\dfrac{2-\dfrac{3}{x}}{\dfrac{2}{x}-3}\right]$

$$\lim\limits_{x\to 0}\left[\dfrac{\dfrac{2x-3}{x}}{\dfrac{2-3x}{x}}\right]=\lim\limits_{x\to 0}\left[\dfrac{2x-3}{2-3x}\right]$$
$$=\dfrac{-3}{2}$$

⑧ 7. For what values of x does $\dfrac{x^2+x-2}{x-1}$ differ from its limit by less than 10^{-3} as $x\to 1$?

$L=\lim\limits_{x\to 1}\dfrac{x^2+x-2}{x-1}$ $\left|f(x)-L\right|<10^{-3}$

$\quad=\lim\limits_{x\to 1}\dfrac{\cancel{(x-1)}(x+2)}{\cancel{(x-1)}}$ $\left|\dfrac{x^2+x-2}{x-1}-3\right|<10^{-3}$

$\quad=\lim\limits_{x\to 1}(x+2)$ $\left|x+2-3\right|<10^{-3}$

$\quad=1+2$ $\left|x-1\right|<10^{-3}$

$\quad=3$ $1-10^{-3}<x<1+10^{-3}$

                                    or $0.999<x<1.001$

Topic 1

_____ marks

Topic 2: Secant and Tangent Lines

(5) 1. Determine the equation of the secant line to the curve defined by $y = x^2 + 2x - 3$ which passes through $x = -1$ and $x = 2$.

$y = x^2 + 2x - 3$

At $x = -1$, $y = (-1)^2 + 2(-1) - 3$

$\qquad = -4$

At $x = 2$, $y = (2)^2 + 2(2) - 3$

$\qquad = 5$

The secant passes through $(-1, -4)$ and $(2, 5)$.

$y - y_1 = m(x - x_1)$, where $m = \dfrac{5 - (-4)}{2 - (-1)} = \dfrac{9}{3} = 3$

$y - 5 = 3(x - 2)$

$y - 5 = 3x - 6$

The equation of the secant is $3x - y - 1 = 0$.

(5) 2. Find the slope of the secant line of the graph $f(x) = 5x^2$ through the points where $x = x_1$ and $x = x_2$.

$f(x) = 5x^2$

$\therefore y = 5x^2$

At $x = x_1$, $y = 5x_1^2$

At $x = x_2$, $y = 5x_2^2$

The slope is $\dfrac{5x_2^2 - 5x_1^2}{x_2 - x_1} = \dfrac{5\left(x_2^2 - x_1^2\right)}{x_2 - x_1}$

$\qquad\qquad = \dfrac{5\left(x_2 + x_1\right)\left(x_2 - x_1\right)}{x_2 - x_1}$

$\qquad\qquad = 5\left(x_2 + x_1\right).$

The slope of the secant line is $5\left(x_2 + x_1\right)$.

3. From first principles, calculate $\frac{dy}{dx}$ for the following:

(8)

a. $y = 2x^2 - 3$

$$f(x) = 2x^2 - 3$$

$$f(x+h) = 2(x+h)^2 - 3$$

$$= 2\left(x^2 + 2hx + h^2\right) - 3$$

$$= 2x^2 + 4hx + 2h^2 - 3$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, \; h \neq 0$$

$$= \lim_{h \to 0} \frac{\left(2x^2 + 4hx + 2h^2 - 3\right) - \left(2x^2 - 3\right)}{h}$$

$$= \lim_{h \to 0} \frac{4hx + 2h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h)}{\cancel{h}}$$

$$= 4x$$

(8)

b. $y = 3x - \dfrac{5}{x}$

$$f(x) = 3x - \frac{5}{x}$$

$$f(x+h) = 3(x+h) - \frac{5}{x+h}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, \; h \neq 0$$

$$= \lim_{h \to 0} \frac{3x + 3h - \dfrac{5}{x+h} - 3x + \dfrac{5}{x}}{h}$$

$$= \lim_{h \to 0} \frac{3hx(x+h) - 5x + 5x + 5h}{x(x+h)h}$$

$$= \lim_{h \to 0} \frac{3x\cancel{h}(x+h) + 5\cancel{h}}{x\cancel{h}(x+h)}$$

$$= \lim_{h \to 0} \frac{3x(x+h) + 5}{x(x+h)}$$

$$\frac{dy}{dx} = \frac{3x^2 + 5}{x^2} \; \text{or} \; 3 + \frac{5}{x^2}$$

③ 4. Find the slope of the tangent to the curve $y = x^3 - 2x + 1$ at $x = 1$.

$$\text{Slope} = \frac{dy}{dx} = 3x^2 - 2$$

$$\text{At } x = 1, \ \frac{dy}{dx} = 3(1)^2 - 2$$

$$= 1$$

Therefore, the slope of the tangent is 1.

5. For the function defined by $y = x^3 + 6x^2 - x^{-5}$, find the following:

③ a. the slope of the curve at $x = -1$

$$y = x^3 + 6x^2 - x^{-5}$$

$$\frac{dy}{dx} = 3x^2 + 12x + 5x^{-6}$$

$$\text{At } x = -1, \ \frac{dy}{dx} = 3(-1)^2 + 12(-1) + \frac{5}{(-1)^6}$$

$$= 3 - 12 + 5$$

$$= -4$$

Thus, -4 is the slope at $x = -1$.

③ b. the equation of the tangent at that point

$$y = (-1)^3 + 6(-1)^2 - (-1)^{-5}$$

$$= -1 + 6 + 1$$

$$= 6$$

The point is $(-1, 6)$.

$$y - 6 = -4(x + 1)$$

$$y - 6 + 4x + 4 = 0$$

$$4x + y - 2 = 0$$

The tangent line is $4x + y - 2 = 0$.

⑦

6. Find the equation of the line tangent to the curve $y = (x^2 + 2x + 4)(x - 2)$ at point $(2, 0)$. Give your answer in the form $Ax + By + C = 0$.

$$y = (x^2 + 2x + 4)(x - 2)$$
$$= x^3 - 2^3$$
$$\therefore y = x^3 - 8$$
$$\frac{dy}{dx} = 3x^2$$

At $(2, 0)$, $\frac{dy}{dx} = 3(2)^2$
$$= 3 \times 4$$
$$= 12$$

Since $y - y_1 = m(x - x_1)$, the equation is as follows:
$$y - 0 = 12(x - 2)$$
$$y = 12x - 24$$
In the form $Ax + By + C = 0$, this becomes $12x - y - 24 = 0$.

⑤

7. Find the equations of the lines tangent to the curve at the points where the graph of $y = x(x - 2)(x + 1)$ cuts the x-axis.

$$y = x(x - 2)(x + 1)$$
$$= x^3 - 1x^2 - 2x$$
The x-intercepts are $(0, 0)$, $(2, 0)$, and $(-1, 0)$.
$$y' = 3x^2 - 2x - 2$$
At $x = 0$, $y' = -2$
$$x = 2, \ y' = 6$$
$$x = -1, \ y' = 3$$

At $(0, 0)$, $\quad m = -2 = \dfrac{y - 0}{x - 0}$
$$y = -2x \qquad \therefore 2x + y = 0$$

At $(2, 0)$, $\quad m = 6 = \dfrac{y - 0}{x - 2}$
$$y = 6x - 12 \qquad \therefore 6x - y - 12 = 0$$

At $(-1, 0)$, $\ m = 3 = \dfrac{y - 0}{x + 1}$
$$y = 3x + 3 \qquad \therefore 3x - y + 3 = 0$$

⑧   8. Determine the equations of the lines tangent to the curves $y = \frac{x^2}{4}$ and $y = \frac{16}{x}$ at the point where the curves intersect.

$$y = \frac{x^2}{4} \quad ①$$

$$y = \frac{16}{x} \quad ②$$

$\frac{x^2}{4} = \frac{16}{x}$ at the point of intersection.

$$x^3 = 64$$

$$x = 4$$

$$y = \frac{16}{x}$$

$$= 4$$

Therefore, the point of intersection is $(4, 4)$.

For $y = \frac{x^2}{4}$, $y' = \frac{x}{2}$.

At $(4, 4)$, $y' = 2$

$$\frac{y - 4}{x - 4} = 2$$

$$y - 4 = 2x - 8$$

$$2x - y - 4 = 0$$

For $y = \frac{16}{x}$, $y' = -16x^{-2}$ or $y' = \frac{-16}{x^2}$.

At $(4, 4)$, $y' = -1$

$$\frac{y - 4}{x - 4} = -1$$

$$y - 4 = -x + 4$$

$$x + y - 8 = 0$$

The two required equations are $2x - y - 4 = 0$ and $x + y - 8 = 0$.

Topic 2

_____ marks